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Vacuum polarization induced by a cosmic string in anti-de Sitter spacetime

E. R. Bezerra de Mello[1]* and A. A. Saharian[1,2]†

[1] *Departamento de Física-CCEN, Universidade Federal da Paraíba 58.059-970, Caixa Postal 5.008, João Pessoa, PB, Brazil*

[2] *Department of Physics, Yerevan State University, 1 Alex Manoogian Street, 0025 Yerevan, Armenia*

Abstract

In this paper we investigate the vacuum expectation values (VEVs) of the field squared and the energy-momentum tensor associated with a massive scalar quantum field induced by a generalized cosmic string in D-dimensional anti-de Sitter (AdS) spacetime. In order to develop this analysis we evaluate the corresponding Wightman function. As we shall observe, this function is expressed as the sum of two terms: the first one corresponds to the Wightman function in pure AdS bulk and the second one is induced by the presence of the string. The second contribution is finite at coincidence limit and is used to provide closed expressions for the parts in the VEVs of the field squared and the energy-momentum tensor induced by the presence of the string. Because the analysis of vacuum polarizations effects in pure AdS spacetime have been developed in the literature, here we are mainly interested in the investigation of string-induced effects. We show that the curvature of the background spacetime has an essential influence on the VEVs at distances larger than the curvature radius. In particular, at large distances the decay of the string-induced VEVs is power-law for both massless and massive fields. The string-induced parts vanish on the AdS boundary and they dominate the pure AdS part for points near the AdS horizon.

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1 Introduction

Symmetry breaking phase transitions in the early universe have several cosmological consequences and provide an important link between particle physics and cosmology. In particular, different types of topological objects may have been formed by the vacuum phase transitions after Planck time [1, 2]. Among them the cosmic strings are of special interest. Although recent observational data on the cosmic microwave background have ruled out cosmic strings as the primary source for primordial density perturbation, they are still candidate for the generation of a number of interesting physical effects such as gamma ray bursts [3], gravitational waves [4] and high energy cosmic rays [5]. Recently, cosmic strings have attracted renewed interest partly

*E-mail: emello@fisica.ufpb.br
†E-mail: saharian@ysu.am

because a variant of their formation mechanism is proposed in the framework of brane inflation [6]-[8].

The gravitational field produced by a cosmic string may be approximated by a planar angle deficit in the two-dimensional sub-space. In quantum field theory the corresponding non-trivial topology induces non-zero vacuum expectation values for physical observables. The analysis of the vacuum polarization effects associated to scalar and fermionic fields have been done in [9]-[15] and [16]-[20], respectively, for the geometry of an idealized cosmic string on background of flat spacetime. For curved backgrounds additional effects appear due to the bulk gravitational field. Recently the combined effects of non-trivial topology and background curvature on the local characteristics of the scalar and fermionic vacua have been investigated in [21, 22] for a cosmic string in de Sitter spacetime. It has been shown that, depending on the curvature radius of de Sitter spacetime, two regimes are realized with monotonic and oscillatory behavior of the vacuum expectation values at distances larger than the de Sitter curvature radius. In this paper we want to continue along similar line of investigation analyzing the vacuum polarization effects associated with a massive scalar quantum field in a high dimensional anti-de Sitter (AdS) spacetime in the presence of a cosmic string. Recently, the geometry of a cosmic string in the background AdS spacetime has been discussed in [23, 24]. In [24], AdS/conformal field theory (CFT) correspondence was used for the calculation of the Green function for a scalar field living on the cone.

AdS spacetime is remarkable from different points of view. The early interest in this spacetime was motivated by the question of principal nature related to the quantization of fields propagating on curved backgrounds. The presence of both regular and irregular modes and the possibility of interesting causal structure lead to a number of new phenomena. The importance of this theoretical work increased when it was discovered that AdS spacetime generically arises as a ground state in extended supergravity and in string theories. Further interest in this subject was generated by the appearance of two models where AdS geometry plays a special role. The first model, the AdS/CFT correspondence (for a review see [25]), represents a realization of the holographic principle and relates string theories or supergravity in the AdS bulk with a conformal field theory living on its boundary. It has many interesting consequences and provides a powerful tool to investigate gauge field theories. The second model is a realization of a braneworld scenario with large extra dimensions and provides a solution to the hierarchy problem between the gravitational and electroweak mass scales (for reviews on braneworld gravity and cosmology see [26, 27]). In this model the main idea to resolve the large hierarchy is that the small coupling of 4-dimensional gravity is generated by the large physical volume of extra dimensions. Braneworlds naturally appear in the string/M theory context and provide a novel setting for discussing phenomenological and cosmological issues related to extra dimensions.

Among the most important characteristics of the vacuum state are the vacuum expectation values (VEVs) of the field squared and the energy-momentum tensor. In the present paper we evaluate these VEVs for a massive scalar field with general curvature coupling parameter in the geometry of a cosmic string on background of AdS spacetime. Though the corresponding operators are local, due to the global nature of the vacuum state these quantities carry an important information about the topology of the spacetime. In addition to describing the physical structure of the quantum field at a given point, the energy-momentum tensor acts as the source in the Einstein equations and therefore plays an important role in modelling self-consistent dynamics involving the gravitational field. As the first step for the investigation of vacuum densities we evaluate the Wightman function. This function gives comprehensive insight into vacuum fluctuations and determines the response of a particle detector of the Unruh-DeWitt type. The problem under consideration is also of separate interest as an example with gravitational and topologically-induced polarizations of the vacuum, where all calculations can

be performed in a closed form.

The paper is organized as follows. In Section 2 we present the background associated with the geometry under consideration and the solution of the Klein-Gordon equation by using *Poincaré* coordinates and admitting an arbitrary curvature coupling. Also we present an integral representation of the Wightman function for an arbitrary planar angle deficit. In Sections 3 and 4 we evaluate the parts in the VEVs of the field squared and the energy-momentum tensor induced by the cosmic string. Finally, the main results are summarized in Section 5. In this paper we shall use the units $\hbar = G = c = 1$.

2 Wightman function

The main objective of this section is to obtain the positive frequency Wightman function associated with a massive scalar field in a higher-dimensional AdS spacetime in presence of a cosmic string. This function is important in the calculation of vacuum polarization effects. In order to do that we first obtain the complete set of normalized mode functions for the Klein-Gordon equation admitting an arbitrary curvature coupling.

In cylindrical coordinates, the geometry associated with a cosmic string in a 4-dimensional AdS spacetime is given by the line element below (considering a static string along the y-axis):

$$ds^2 = e^{-2y/a}(-dt^2 + dr^2 + r^2 d\phi^2) + dy^2 \ , \tag{1}$$

where $r \geqslant 0$ and $\phi \in [0,\ 2\pi/q]$ define the coordinates on the conical geometry, $(t,\ y) \in (-\infty,\ \infty)$, and the parameter a determines the curvature scale of the background spacetime. The parameter q is bigger than unity and codifies the presence of the cosmic string. By using *Poincaré* coordinate defined by $z = ae^{y/a}$, the line element above is written in the form conformally related to the line element associated with a cosmic string in Minkowski spacetime:

$$ds^2 = (a/z)^2(-dt^2 + dr^2 + r^2 d\phi^2 + dz^2) \ . \tag{2}$$

For the new coordinate one has $z \in [0,\ \infty)$. Limiting values $z = 0$ and $z = \infty$ correspond to the AdS boundary and horizon, respectively.

For an infinite straight cosmic string in the background of Minkowski spacetime the line element (expression inside the brackets of the right-hand side of (2)) has been derived in [28] by making use of two approximations: the weak-field approximation and the thin-string one. In this case the parameter q is related to the mass per unit length μ of the string by the formula $1/q = 1 - 4G\mu$, where G is the Newton's gravitational constant. In the standard scenario of the cosmic string formation in the early universe one has $G\mu \sim (\eta/m_{\rm Pl})^2$, where η is the energy scale of the phase transition at which the string is formed and $m_{\rm Pl}$ is the Planck mass. For GUT scale strings $G\mu \ll 1$ and the weak-field approximation is well justified. The validity of the line element with the planar angle deficit has been extended beyond linear perturbation theory by several authors [29] (see also [2]). In this case the parameter q need not to be close to 1. An interesting limiting case with $q \gg 1$ has been discussed in [2]. Note that in braneworld scenarios based on AdS spacetime, to which the results given below in the present paper could be applied, the fundamental Planck scale is much smaller than $m_{\rm Pl}$ and can be of the order of string formation energy scale. Similar to the string in Minkowski spacetime, the line element (1) is an exact solution of the Einstein equation in the presence of negative cosmological constant and the string [23, 24], for arbitrary value of q.

Note that conical defects with the parameter q essentially different from unity appear also in a number of condensed matter systems. An example is the graphitic cone, the long-wavelength electronic properties of which are well described by a Dirac-like model for electronic states in

graphene. Graphitic cones are obtained from the graphene sheet if one or more sectors are excised. The opening angle of the cone is related to the number of sectors removed, N, by the formula $2\pi(1-N/6)$, with $N=1,2,\ldots,5$. All these angles have been observed in experiments [30].

The generalization of (2) to D-dimensional AdS spacetimes is done in the usual way, by adding extra Euclidean coordinates:

$$ds^2=(a/z)^2(-dt^2+dr^2+r^2d\phi^2+dz^2+\sum_{i=1}^{d}dz_i^2)\ , \tag{3}$$

where $d=D-4$. Note that the curvature scale a is related to the cosmological constant, Λ, and the Ricci scalar, R, by the formulas

$$\Lambda=-\frac{(D-1)(D-2)}{2a^2},\quad R=-\frac{D(D-1)}{a^2}\ . \tag{4}$$

The field equation that will be considered is

$$(\nabla_\mu\nabla^\mu-m^2-\xi R)\Phi(x)=0\ , \tag{5}$$

where ξ is a curvature coupling constant. In the coordinate system defined by (3), the complete set of solutions to this equation, regular at the boundary $z=0$, is characterized by the set of quantum number $\sigma=(\lambda,\ n,\ p,\mathbf{k})$ and is given by:

$$\Phi_\sigma(x)=\sqrt{\frac{qa^{2-D}p\lambda}{2(2\pi)^{D-3}E}}\ \ z^{(D-1)/2}J_\nu(\lambda z)J_{|n|q}(pr)e^{i(nq\phi+\mathbf{k}\cdot\mathbf{x}-Et)}\ , \tag{6}$$

with $\mathbf{x}=(z,z_1,\ldots,z_d)$,

$$(\lambda,\ p)\in[0,\infty)\ ,\ k_i\in(-\infty,\ \infty),\ n=0,\pm1,\pm2,\ ... \tag{7}$$

In (6), $J_\mu(u)$ represents the Bessel function, $E=\sqrt{\lambda^2+p^2+\mathbf{k}^2}$, and

$$\nu=\sqrt{(D-1)^2/4-\xi D(D-1)+m^2a^2}. \tag{8}$$

The mode functions in (6) are normalized by the standard Klein-Gordon orthonormalization condition

$$i\int d^{D-1}x\sqrt{|g|}g^{00}[\Phi_\sigma(x)\partial_t\Phi^*_{\sigma'}(x)-\Phi^*_{\sigma'}(x)\partial_t\Phi^*_\sigma(x)]=\delta_{\sigma,\sigma'}\ , \tag{9}$$

where the integral is evaluated over the spatial hypersurface $t=$ const, and $\delta_{\sigma,\sigma'}$ represents the Kronecker-delta for discrete indices and the Dirac delta function for continuous ones.

We now employ the mode-sum formula to evaluate the positive frequency Wightman function:

$$G(x,x')=\sum_\sigma\Phi_\sigma(x)\Phi^*_\sigma(x')\ . \tag{10}$$

Substituting (6) into (10) we obtain

$$\begin{aligned}G(x,x')\ &=\ \frac{q\ (zz')^{(D-1)/2}}{2(2\pi)^{D-3}\ a^{D-2}}\sum_{n=-\infty}^{\infty}e^{inq\Delta\phi}\int d\mathbf{k}\int_0^\infty dp\ p\int_0^\infty d\lambda\ \frac{\lambda}{E}\\ &\times\ e^{i\mathbf{k}\cdot\Delta\mathbf{x}-iE\Delta t}J_{|n|q}(pr)J_{|n|q}(pr')J_\nu(\lambda z)J_\nu(\lambda z')\ ,\end{aligned} \tag{11}$$

where $\Delta t = t - t'$ and $\Delta \mathbf{x} = \mathbf{x} - \mathbf{x}'$. In order to provide a more workable expression for the Wightman function, after a Wick rotation, $i\Delta t = \Delta\tau$, we use the identity:

$$\frac{e^{-\sqrt{\lambda^2+p^2+\mathbf{k}^2}\Delta\tau}}{\sqrt{\lambda^2+p^2+\mathbf{k}^2}} = \frac{2}{\sqrt{\pi}}\int_0^\infty ds\ e^{-s^2(\lambda^2+p^2+\mathbf{k}^2)-\Delta\tau^2/4s^2} \ . \tag{12}$$

Now with the help of [31] and after some intermediate steps, we can express the Wightman function in an integral representation as shown below:

$$G(x,x') = \frac{q}{a^{D-2}}\left(\frac{zz'}{4\pi}\right)^{(D-1)/2}\int_0^\infty \frac{ds}{s^D}\ e^{-(\mathcal{V}^2/4s^2)}S_q\left(rr'/(2s^2)\right)I_\nu\left(zz'/(2s^2)\right) \ , \tag{13}$$

where $I_\nu(u)$ is the modified Bessel function,

$$S_q(u) = \sum_{n=-\infty}^{\infty} e^{inq\Delta\phi}I_{|n|q}(u) = 2{\sum_{n=0}^{\infty}}' \cos(nq\Delta\phi)I_{nq}(u) \tag{14}$$

and

$$\mathcal{V}^2 = r^2 + r'^2 + z^2 + z'^2 + \Delta\mathbf{x}^2 - \Delta t^2 \ . \tag{15}$$

In (14) the prime on the sign of summation means that the term $n = 0$ should be halved.

In [32] (see also [33]), a general expression for the summation (14) is presented in terms of an integral representation:

$$\begin{aligned} S_q(u) &= \frac{1}{q}\sum_k e^{u\cos(2\pi k/q+\Delta\phi)} - \frac{1}{2\pi}\sum_{j=\pm1}\sin(q\pi + jq\Delta\phi) \\ &\quad\times\int_0^\infty dx \frac{e^{-u\cosh x}}{\cosh(qx)-\cos(q\pi+jq\Delta\phi)} \ , \end{aligned} \tag{16}$$

where k is an integer number defined by

$$-q/2 + q\Delta\phi/(2\pi) \leqslant k \leqslant q/2 + q\Delta\phi/(2\pi). \tag{17}$$

We can see that, for integer values of q, formula (16) reduces to the well-known result [34, 35]:

$$2{\sum_{n=0}^{\infty}}' \cos(nq\Delta\phi)I_{nq}(u) = \frac{1}{q}\sum_{k=0}^{q-1} e^{u\cos(2k\pi/q-\Delta\phi)}. \tag{18}$$

Substituting (16) into (13), with the help of [31] and after some steps, we obtain:

$$\begin{aligned} G(x,x') &= \frac{a^{2-D}}{(2\pi)^{D/2}}\left[\sum_k F_\nu(u_k) - \frac{q}{2\pi}\sum_{j=\pm1}\sin(q\pi+jq\Delta\phi)\right. \\ &\quad\left.\times\int_0^\infty dx\ \frac{F_\nu(u_x)}{\cosh(qx)-\cos(q\pi+jq\Delta\phi)}\right] \ , \end{aligned} \tag{19}$$

where the summation over k goes in accordance with (17) and we have introduced a new function

$$F_\nu(u) = \frac{e^{-i\pi(D/2-1)}Q_{\nu-1/2}^{D/2-1}(u)}{(u^2-1)^{(D-2)/4}} \ . \tag{20}$$

Here, $Q_\mu^\nu(u)$ is the associated Legendre function, whose arguments in (19) are given by

$$
\begin{aligned}
u_k &= 1+\frac{r^2+r'^2-2rr'\cos(\Delta\phi-2\pi k/q)+(\Delta z)^2+(\Delta\mathbf{x})^2-\Delta t^2}{2zz'}, \\
u_x &= 1+\frac{r^2+r'^2+2rr'\cosh x+(\Delta z)^2+(\Delta\mathbf{x})^2-\Delta t^2}{2zz'} .
\end{aligned} \tag{21}
$$

The component $k=0$ of the first term of (19) coincides with the Wightmann function in a pure AdS spacetime, $G_{\mathrm{AdS}}(x,x')$. Because the analysis of vacuum polarization effects in AdS spacetime in the absence of the string has been developed in the literature by many authors, here we are mostly interested with the vacuum quantum effects induced by the cosmic string. Writing the above Wightman function in the decomposed form

$$
G(x,x')=G_{\mathrm{AdS}}(x,x')+G_c(x,x') \ , \tag{22}
$$

we shall consider in our future analysis the function $G_c(x,x')$, which corresponds to the correction in the Wightman function introduced by the cosmic string. Moreover, because the presence of the string does not modify the local geometry for $r\neq 0$, this component of the Wigthmann function is finite at the coincidence limit.

We can express the function $F_\nu(z)$ in (20) in terms of the hypergeometric function as:

$$
F_\nu(u)=\frac{B_\nu}{u^{\beta_\nu}}F(\beta_\nu/2+1/2,\beta_\nu/2;\nu+1;u^{-2}), \tag{23}
$$

where we use the notations

$$
\begin{aligned}
\beta_\nu &= \nu+(D-1)/2, \\
B_\nu &= \frac{\sqrt{\pi}\Gamma(\beta_\nu)}{2^{\nu+1/2}\Gamma(\nu+1)}.
\end{aligned} \tag{24}
$$

In the discussion below, we shall need the asymptotics of the function $F_\nu(u)$ for large values of the argument and for u close to 1. For large values of u one has

$$
F_\nu(u)\approx B_\nu/u^{\beta_\nu}. \tag{25}
$$

For u close to 1 we use the linear transformation formula 15.3.6 from [36]. In the leading order this gives:

$$
F_\nu(u)\approx\frac{\Gamma(D/2-1)}{2(u-1)^{D/2-1}}. \tag{26}
$$

For a conformally coupled massless scalar field we have $\nu=1/2$. By using the formula 15.1.10 from [36] for the hypergeometric function in (23), we find

$$
F_\nu(u)=-\frac{1}{2}\Gamma(D/2-1)\left[(u+1)^{1-D/2}-(u-1)^{1-D/2}\right]. \tag{27}
$$

The corresponding Wightman function is presented in the form

$$
\begin{aligned}
G(x,x') &= \frac{\Gamma(D/2-1)\ (zz')^{D/2-1}}{4\pi^{D/2}a^{D-2}}\left[\sum_k(u_k^{(-)1-D/2}-u_k^{(+)1-D/2})\right. \\
&\left.-\frac{q}{2\pi}\sum_{j=\pm1}\sin(q\pi+jq\Delta\phi)\int_0^\infty dx\ \frac{u_x^{(-)1-D/2}-u_x^{(+)1-D/2}}{\cosh(qx)-\cos(q\pi+jq\Delta\phi)}\right] ,
\end{aligned} \tag{28}
$$

with the notations

$$\begin{aligned} u_k^{(\pm)} &= r^2+r'^2-2rr'\cos(\Delta\phi-2\pi k/q)+\left(z\pm z'\right)^2+(\Delta\mathbf{x})^2-\Delta t^2, \\ u_x^{(\pm)} &= r^2+r'^2+2rr'\cosh x+\left(z\pm z'\right)^2+(\Delta\mathbf{x})^2-\Delta t^2. \end{aligned} \tag{29}$$

We could obtain expression (28) by using the conformal relation of the problem under consideration with the problem of a cosmic string in background of the Minkowski spacetime with a Dirichlet boundary perpendicular to the string and located at $z=0$ (vacuum polarization effects in the latter geometry have been investigated in [37] for a massive scalar field with general curvature coupling parameter). The presence of the boundary in the Minkowskian counterpart is related to the fact that for the geometry of a string in AdS spacetime we have chosen regular mode functions (6). For a conformally coupled massless field these functions are conformally related to the mode functions for the string in Minkowski spacetime which obey the Dirichlet boundary condition at $z=0$.

3 Calculation of $\langle\phi^2\rangle$

In this section and in the following, we shall investigate the vacuum polarization effects induced by the cosmic string in AdS spacetime. Two main calculations will be performed. The evaluation of the VEV of the field squared, in the first place, followed by the evaluation of the VEV of the energy-momentum tensor.

Formally the evaluation of the VEV of the field squared is given taking the coincidence limit of the arguments in the expression for the Wightman function:

$$\langle\phi^2\rangle = \lim_{x'\to x} G(x,x') \ . \tag{30}$$

The coincidence limit in the right-hand side is divergent and some renormalization procedure is needed. By taking into account the decomposition of the Wightman function, given by (22), we can see that for $r\neq 0$ the divergences come from the pure AdS part. The part in the Wightman function induced by the presence of the cosmic string is finite in the coincidence limit. Of course, we could expect this result from general arguments. The divergences in the expectation values of local physical observables, like field squared and energy-momentum tensor, are entirely geometrical (for a general discussion see [38]). For expectation values at a given spacetime point they depend on the curvature tensor and its contractions at the same point. The presence of the cosmic string does not change the local geometry of AdS spacetime for points away from the string and, consequently, the structure of the divergences remains the same as in pure AdS spacetime. In particular, they do not depend on the parameters of the cosmic string. In accordance with the standard procedure, for the renormalization of the VEV in (30) we subtract from the Wightman function in the right-hand side the corresponding DeWitt-Schwinger expansion truncated at the adiabatic order $D+1$. The subtracted terms are determined by the local geometry and they are not affected by the presence of the string. After the subtraction, the coincidence limit is finite. Of course, we can still add finite renormalization terms. But, again, these terms are determined entirely by the local geometry of the spacetime. They do not depend on the planar angle deficit and renormalize the pure AdS part only. The topological parts in the VEV induced by the string, which are the main subject of the present research, are not touched by both the infinite and finite renormalization procedures.

As we have discussed above, the VEV of the field squared given in (30), is presented in a decomposed form:

$$\langle\phi^2\rangle = \langle\phi^2\rangle_{\rm AdS} + \langle\phi^2\rangle_c \ , \tag{31}$$

where $\langle\phi^2\rangle_{\mathrm{AdS}}$ is the VEV in AdS spacetime in the absence of the cosmic string and $\langle\phi^2\rangle_c$ is the string-induced part. Moreover, the renormalization procedure is needed only for the first contribution in the right hand-side of (31). The second contribution is finite at the coincidence limit for $r \neq 0$. Due to the maximal symmetry of the AdS spacetime and the vacuum state under consideration, the renormalized VEV of the field squared, $\langle\phi^2\rangle_{\mathrm{AdS}}$, does not present dependence on the specific point of the spacetime (for the investigation of the VEVs in AdS spacetime see [39]-[43]). So, as we shall see, the contribution induced by the string, $\langle\phi^2\rangle_c$, will become more important than $\langle\phi^2\rangle_{\mathrm{AdS}}$ for points near the string or near the AdS horizon.

According to the discussion of the previous section, here we shall analyze the VEV of the field squared induced by the cosmic string only. This contribution is directly obtained from (19), omitting the $k=0$ term and taking the coincidence limit. Note that in the coincidence limit, u_k in (19) is an even function of k and the summation over k in the range $-q/2 \leqslant k \leqslant q/2$ can be transformed into the summation over the positive values only. Changing also the integration variable, the string-induced part is presented in the form

$$\langle\phi^2\rangle_c = \frac{2a^{2-D}}{(2\pi)^{D/2}}\left[\sum_{k=1}^{[q/2]} F_\nu(w_k) - \frac{q}{\pi}\int_0^\infty dx\, \frac{\sin(q\pi)F_\nu(w_x)}{\cosh(2qx)-\cos(q\pi)}\right] , \tag{32}$$

where $[q/2]$ is the integer part of $q/2$, and

$$w_k = 1+2\rho^2 s_k^2, \quad w_x = 1+2\rho^2\cosh^2 x, \tag{33}$$

with

$$\rho = r/z\ , \quad s_k = \sin(\pi k/q)\ . \tag{34}$$

We observe that the string-induced part is a function of the ratio r/z which is the proper distance from the string, ar/z, measured in units of the AdS curvature radius a. This property is a consequence of the maximal symmetry of AdS spacetime.

For a conformally coupled massless scalar field, by taking into account the expression (28) for the Wightman function, one gets

$$\langle\phi^2\rangle_c = (z/a)^{D-2}\left[\langle\phi^2\rangle_c^{(\mathrm{M})} + \langle\phi^2\rangle_{c,b}^{(\mathrm{M})}\right], \tag{35}$$

where

$$\langle\phi^2\rangle_c^{(\mathrm{M})} = \frac{2\Gamma(D/2-1)}{(4\pi)^{D/2}r^{D-2}}g_{D-2}(q), \tag{36}$$

is the VEV for the boundary-free string geometry in background of flat spacetime and

$$g_n(q) = \sum_{k=1}^{[q/2]} s_k^{-n} - \frac{q}{\pi}\int_0^\infty dx\, \frac{\sin(q\pi)\cosh^{-n}x}{\cosh(2qx)-\cos(q\pi)}. \tag{37}$$

In (35), the term

$$\begin{aligned}\langle\phi^2\rangle_{c,b}^{(\mathrm{M})} &= -\frac{2\Gamma(D/2-1)}{(4\pi)^{D/2}}\left[\sum_{k=1}^{[q/2]}\left(r^2 s_k^2+z^2\right)^{1-D/2}\right.\\ &\left.-\frac{q}{\pi}\sin(q\pi)\int_0^\infty dx\, \frac{\left(r^2\cosh^2 x+z^2\right)^{1-D/2}}{\cosh(2qx)-\cos(q\pi)}\right],\end{aligned} \tag{38}$$

is the part induced by a flat boundary located at $z=0$ in the latter geometry. Closed expressions for $g_n(q)$ can be given for even values of n. In particular, one has

$$g_2(q)=\frac{q^2-1}{6},\ g_4(q)=\frac{\left(q^2-1\right)\left(q^2+11\right)}{90}. \tag{39}$$

For general $q>1$ the function $g_n(q)$ is positive. In [37] we have evaluated the VEV of the field squared in a D-dimensional cosmic string spacetime, associated with scalar quantum field obeying Dirichlet or Neumann boundary conditions on a flat surface orthogonal to the string. The result found for this VEV induced by the string, considering the Dirichlet boundary and massless field, is conformally related to $\langle\phi^2\rangle_c$ with the conformal factor $(z/a)^{D-2}$ (see (35)).

Let us discuss the asymptotics of the VEV of the field squared near the string and at large distances. For points near the string, $\rho\ll 1$, by using the asymptotic expression (26), in the leading order we find

$$\langle\phi^2\rangle_c\approx\frac{2\,\Gamma(D/2-1)}{(4\pi)^{D/2}(ar/z)^{D-2}}g_{D-2}(q)\ . \tag{40}$$

Comparing with (36), we see that the expression on the right of (40) coincides with the corresponding expression for the VEV of the field squared in the geometry of a string in background of Minkowski spacetime, where the distance from the string is replaced by the proper distance ar/z. As the pure AdS part in the VEV, $\langle\phi^2\rangle_{\mathrm{AdS}}$, is a constant, we conclude that near the string the topological part dominates in the total VEV. For a fixed value of the radial coordinate r, the limit under consideration corresponds to large values of z, i.e., to points near the AdS horizon. Hence, we conclude that near the horizon the string-induced part behaves as z^{D-2}.

At large distances from the string, $\rho\gg 1$, we use the asymptotic expression (25). In the leading order this gives:

$$\langle\phi^2\rangle_c\approx\frac{a^{2-D}\ B_\nu g_{2\beta_\nu}(q)}{2^{\beta_\nu-1}(2\pi)^{D/2}(r/z)^{2\nu+D-1}}\ . \tag{41}$$

At large distances the total VEV is dominated by the pure AdS part. As it is seen, the decay of the topological part of the field squared with the distance is power-law for both massless and massive fields. Note that for a string in background of the Minkowski spacetime the decay of the VEV at large distances is power-law for a massless field and exponential for a massive field. Hence, we see that the curvature of the background spacetime has an essential influence on the VEVs at distances larger than the curvature radius of the background spacetime. From (41) it follows that, for fixed values of r, the string-induced part vanishes on the AdS boundary as $z^{2\nu+D-1}$.

It can be seen that for integer value of q, the expression (32) takes a simpler form:

$$\langle\phi^2\rangle_c=\frac{a^{2-D}}{(2\pi)^{D/2}}\sum_{k=1}^{q-1}F_\nu(w_k)\ . \tag{42}$$

Note that, for even values of q the integral term in (32) contributes as well. Indeed, we assume that $q=2n+\epsilon$ with $\epsilon\to 0$. As the integral is divergent at the lower limit, the dominant contribution for small ϵ comes from the region near this limit (small values of x). Expanding the integrand, we can see that the second term in the square brackets of (32) gives the contribution $(1/2)F_\nu(1+2\rho^2)$. In particular, for a conformally coupled massless field the formula (42) reduces to

$$\langle\phi^2\rangle_c=\frac{2\Gamma(D/2-1)}{(4\pi)^{D/2}a^{D-2}}\sum_{k=1}^{q-1}\left[\frac{1}{(\rho s_k)^{D-2}}-\frac{1}{\left(1+\rho^2 s_k^2\right)^{D/2-1}}\right]. \tag{43}$$

For even values of D, the sum with the first term in the square brackets of (43) can be further simplified by using the recurrence relation given in [14] and the result $\sum_{k=1}^{q-1} s_k^{-2} = (q^2-1)/3$. So, in the 4-dimensional spacetime we may write

$$\langle \phi^2 \rangle_c = \frac{(z/a)^2}{16\pi^2} \left(\frac{q^2-1}{3r^2} - \sum_{k=1}^{q-1} \frac{1}{z^2 + r^2 s_k^2} \right) . \tag{44}$$

For a six-dimensional spacetime, the summation needed is: $\sum_{k=1}^{q-1} \sin^{-4}(\pi k/q) = (q^2-1)(q^2+11)/45$. As a result, for the VEV we obtain:

$$\langle \phi^2 \rangle_c = \frac{(z/a)^4}{64\pi^3} \left[\frac{(q^2-1)(q^2+11)}{45r^4} - \sum_{k=1}^{q-1} \frac{1}{(z^2 + r^2 s_k^2)^2} \right] . \tag{45}$$

In figure 1 we have displayed the dependence of the string-induced part in the VEV of the field squared versus the proper distance from the string (measured in units of the AdS curvature radius). The graphs are plotted for $D = 4$ minimally (full curves) and conformally (dashed curves) coupled massless scalar fields for separate values of the parameter q (numbers near the curves).

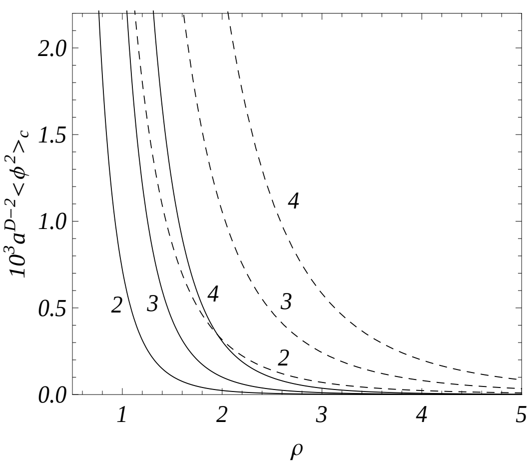

Figure 1: The string-induced part in the VEV of the field squared as a function of the proper distance from the string for $D = 4$ minimally (full curves) and conformally (dashed curves) massless scalar fields. The numbers near the curves correspond to the values of the parameter q.

4 VEV of the energy-momentum tensor

Similar to the case of the field squared, the VEV of the energy-momentum tensor is presented in the decomposed form:

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu} \rangle_{\mathrm{AdS}} + \langle T_{\mu\nu} \rangle_c , \tag{46}$$

where the first and second terms on the right-hand side correspond to the purely AdS and string-induced parts, respectively. Because of the maximal symmetry of AdS spacetime and the vacuum state under consideration, the pure AdS part in the VEV of the energy-momentum tensor is proportional to the metric tensor: $\langle T_{\mu\nu} \rangle_{AdS} = g_{\mu\nu} \langle T_\beta^\beta \rangle_{\mathrm{AdS}}/D$, with $\langle T_\beta^\beta \rangle_{AdS}$ being the

corresponding trace (the expression for the latter in arbitrary number of spacetime dimensions can be found in [43]). In this section we shall analyze the contribution induced by the string. To develop this calculation we shall use the following expression:

$$\langle T_{\mu\nu}\rangle_c = \lim_{x'\to x}\partial_{\mu'}\partial_\nu G_c(x,x') + \left[(\xi-1/4)\,g_{\mu\nu}\nabla_\alpha\nabla^\alpha - \xi\nabla_\mu\nabla_\nu + \xi R_{\mu\nu}\right]\langle\phi^2\rangle_c \ , \tag{47}$$

where for the AdS spacetime, the Ricci tensor reads

$$R_{\mu\nu} = -(D-1)g_{\mu\nu}a^{-2} \ . \tag{48}$$

We may write the covariant d'Alembertian of $\langle\phi^2\rangle_c$, appearing in (47), as shown below:

$$\nabla_\alpha\nabla^\alpha\langle\phi^2\rangle_c = \frac{8a^{-D}}{(2\pi)^{D/2}}\left[\sum_{k=1}^{[q/2]} f(w_k,s_k) - \frac{q}{\pi}\int_0^\infty dx\frac{\sin(q\pi)f(w_x,\cosh x)}{\cosh(2qx)-\cos(q\pi)}\right] \ , \tag{49}$$

with the notation

$$f(u,v) = (u-1)\left(2v^2+u-1\right)F''_\nu(u) + [2v^2 + \frac{D+1}{2}(u-1)]F'_\nu(u), \tag{50}$$

and the prime means the derivative of the functions with respect to their arguments. By using the formula 15.2.3 for the hypergeometric function from [36], the derivatives appearing in (50) are written in the form

$$\begin{aligned} F'_\nu(u) &= -\frac{\beta_\nu B_\nu}{u^{\beta_\nu+1}}F(\beta_\nu/2+1/2,\beta_\nu/2+1;c;u^{-2}), \\ F''_\nu(u) &= \frac{\beta_\nu\left(\beta_\nu+1\right)B_\nu}{u^{\beta_\nu+2}}F(\beta_\nu/2+3/2,\beta_\nu/2+1;c;u^{-2}). \end{aligned} \tag{51}$$

By using the the expression for the Wightman function and the VEV of the field squared, after a long but straightforward calculations, the diagonal components of the energy-momentum tensor are presented in the form (no summation over μ):

$$\langle T_\mu^\mu\rangle_c = \frac{2a^{-D}}{(2\pi)^{D/2}}\left[\sum_{k=1}^{[q/2]} G_\mu(w_k,s_k) - \frac{q}{\pi}\int_0^\infty dx\ \frac{\sin(q\pi)G_\mu(w_x,\cosh x)}{\cosh(2qx)-\cos(q\pi)}\right] \ , \tag{52}$$

where s_k is given in (34),

$$G_\mu(u,v) = f_\mu(u,v) + (4\xi-1)f(u,v) - \xi(D-1)F_\nu(u), \tag{53}$$

and

$$\begin{aligned} f_0(u,v) &= -[1+2\xi(u-1)]F'_\nu(u) \ , \\ f_1(u,v) &= [2v^2\left(1-2\xi\right)-1-2\xi(u-1)]F'_\nu(u) + 2v^2(1-4\xi)(u-1)F''_\nu(u) \ , \\ f_2(u,v) &= -[1+2v^2\left(2\xi-1\right)+2\xi(u-1)]F'_\nu(u) - 2(u-1)(1-v^2)F''_\nu(u) \ , \\ f_3(u,v) &= [(1-4\xi)\left(u-1\right)-1]F'_\nu(u) + (u-1)^2(1-4\xi)F''_\nu(u) \ . \end{aligned} \tag{54}$$

For the components $\langle T_\mu^\mu\rangle_c$ with $\mu = 4\ ...\ D-1$, one has $\langle T_\mu^\mu\rangle_c = \langle T_0^0\rangle_c$. The latter relation is a direct consequence of the invariance of the problem with respect to the boosts along the corresponding directions. The last two terms in the right-hand side of (53) are the same for all diagonal components and come from the first end last terms in the square brackets of (47).

Note that the string-induced part in the energy density is given by $-\langle T_0^0\rangle_c$. For the non-zero off-diagonal component we have,

$$\langle T_3^1\rangle_c = \frac{2a^{-D}}{(2\pi)^{D/2}\rho}\left[\sum_{k=1}^{[q/2]} G(w_k) - \frac{q}{\pi}\int_0^\infty dx\, \frac{\sin(q\pi)G(w_x)}{\cosh(2qx)-\cos(q\pi)}\right] , \tag{55}$$

where

$$G(u) = (u-1)\left[(u-1)(4\xi-1)F_\nu''(u) + (2\xi-1)F_\nu'(u)\right] . \tag{56}$$

It can be explicitly checked that the string-induced parts in the VEVs obey the trace relation

$$\langle T_\mu^\mu\rangle_c = D(\xi-\xi_D)\nabla_\alpha\nabla^\alpha\langle\phi^2\rangle_c + m^2\langle\phi^2\rangle_c. \tag{57}$$

In particular, the topological part in the VEV of the energy-momentum tensor is traceless for a conformally coupled massless scalar field. The trace anomaly appears in the pure AdS part only. Note that for a $D=4$ conformally coupled massless scalar field (see, for example, [41]) $\langle T_\mu^\mu\rangle_{\rm AdS} = -1/(240\pi^2a^4)$. We have also observed that for a massless conformally coupled field, the above expressions are conformally related with the corresponding ones obtained in [37] with the conformal factor $(z/a)^D$. In particular, for the off-diagonal component we have

$$\begin{aligned}\langle T_3^1\rangle_c \;=\; & -\frac{2\Gamma(D/2+1)\rho}{(4\pi)^{D/2}(D-1)a^D}\Bigg[\sum_{k=1}^{[q/2]}\frac{s_k^2}{(1+\rho^2 s_k^2)^{D/2+1}} \\ & -\frac{q}{\pi}\sin(q\pi)\int_0^\infty dx\,\frac{(1+\rho^2\cosh^2 x)^{-D/2-1}}{\cosh(2qx)-\cos(q\pi)}\cosh^2 x\Bigg].\end{aligned} \tag{58}$$

Note that in this case the off-diagonal component vanishes on the string. As it will be seen below, the diagonal components diverge on the string.

Due to the presence of the off-diagonal component, from the covariant conservation equation for the energy-momentum tensor, $\nabla_\mu\langle T_\nu^\mu\rangle_c = 0$, two non-trivial differential equations follow:

$$\frac{1}{r}\partial_r\left(r\langle T_1^1\rangle_c\right) - \frac{1}{r}\langle T_2^2\rangle_c - \frac{D}{z}\langle T_1^3\rangle_c + \partial_z\langle T_1^3\rangle_c = 0 \tag{59}$$

and

$$\frac{1}{r}\partial_r\left(r\langle T_3^1\rangle_c\right) - \frac{D}{z}\langle T_3^3\rangle_c + \partial_z\langle T_3^3\rangle_c + \frac{1}{z}\langle T_\mu^\mu\rangle_c = 0 . \tag{60}$$

It can be explicitly checked that the above relations are obeyed by the string-induced parts in the VEV of the energy-momentum tensor, given by expressions (52) and (55).

Relatively simple expressions for the VEV of the energy-momentum tensor are obtained near the string and at large distances. First we consider the region near the string, assuming that $\rho \ll 1$. By taking into account (26), in the leading order for the diagonal components we find (no summation over μ):

$$\langle T_\mu^\mu\rangle_c \approx -\frac{\Gamma(D/2)}{(4\pi)^{D/2}(a\rho)^D}\left[f_\mu^{(0)}g_D(q) + f_\mu^{(1)}g_{D-2}(q)\right], \tag{61}$$

with the notations

$$\begin{aligned} f_\mu^{(0)} &= -f_2^{(0)}/(D-1) = -1, \quad \mu\neq 2, \\ f_0^{(1)} &= (D-2)(1-4\xi), \quad f_1^{(1)} = -f_2^{(1)}/(D-1) = 4\xi, \end{aligned} \tag{62}$$

and $f_{\mu}^{(j)} = f_{0}^{(j)}$ for $\mu \geqslant 3$, $j = 1, 2$. The leading term given by (61) coincides with the corresponding expression for a string in Minkowski spacetime with the distance from the string replaced by ar/z. For the off-diagonal component the leading term is given by the expression

$$\langle T_{3}^{1}\rangle_{c} \approx (\xi - \xi_{D})\,\frac{4(D-1)\Gamma(D/2)}{(4\pi)^{D/2}a(ar/z)^{D-1}}g_{D-2}(q)\ , \tag{63}$$

with the function $g_{n}(q)$ defined in (37). Note that the off-diagonal component is suppressed by an additional factor r/z compared with the diagonal ones. For a conformally coupled massless field the behavior of the off-diagonal component near the string directly follows from (58). In this case the off-diagonal component vanishes on the string. For $D = 4$, in (61) and (63) we can use the expressions (39). In this case it can be seen that for a conformally coupled scalar field the energy density near the string is negative. For a minimally coupled field the energy density is positive for $q^{2} < 19$ and negative for $q^{2} > 19$. For fixed values of r, the limit under consideration corresponds to points near the AdS horizon. In particular, we see that on the horizon the VEVs of the diagonal components of the energy-momentum tensor diverge as z^{D}. The asymptotic expressions show that near the string or near the horizon the topological part is large and the back-reaction of the quantum effects on the bulk geometry is important.

Now we turn to the asymptotic of the string-induced part at large distances from the string, $\rho \gg 1$. By using the asymptotic expression (25), to the leading order for the diagonal components we get (no summation over μ):

$$\langle T_{\mu}^{\mu}\rangle_{c} \approx -\frac{B_{\nu}\nu\left[\beta_{\nu} - 2\xi\left(2\nu + D\right)\right]}{2^{\beta_{\nu}-1}(2\pi)^{D/2}a^{D}(r/z)^{2\nu+D-1}}g_{2\beta_{\nu}}(q), \tag{64}$$

for $\mu \neq 3$, and

$$\langle T_{3}^{3}\rangle_{c} \approx -\frac{D-1}{2\nu}\langle T_{0}^{0}\rangle_{c}. \tag{65}$$

For both minimally and conformally coupled fields the energy density corresponding to (64) is positive. For the off-diagonal component one has

$$\langle T_{3}^{1}\rangle_{c} \approx \frac{\beta_{\nu}z}{\nu r}\langle T_{0}^{0}\rangle_{c}. \tag{66}$$

The latter is suppressed by an additional factor z/r compared with the diagonal components. Similar to the case of the field squared, the decay of the topological part in the VEV of the energy-momentum tensor is power-law for both massless and massive fields. Note that, at large distances, the stresses in the subspace perpendicular to the z-axis are isotropic. For a minimally coupled massless scalar field, the relation (65) between the energy density and the stress along the z-axis is of the cosmological constant type. Note that, for a fixed value of the radial coordinate, the limit under consideration corresponds to points near the AdS boundary. From the asymptotic expressions we see that the string-induced parts in the diagonal components vanish on the AdS boundary as $z^{2\nu+D-1}$.

In the case of integer q, the general formulas (52) and (55) take the form (no summation over μ)

$$\begin{aligned} \langle T_{\mu}^{\mu}\rangle_{c} &= \frac{a^{-D}}{(2\pi)^{D/2}}\sum_{k=1}^{q-1}G_{\mu}(w_{k}, s_{k})\ , \\ \langle T_{3}^{1}\rangle_{c} &= \frac{a^{-D}}{(2\pi)^{D/2}}\sum_{k=1}^{q-1}G(w_{k}, s_{k})\ , \end{aligned} \tag{67}$$

with functions $G_\mu(u,v)$ and $G(u,v)$ defined in (53) and (56). For the general situation, the complete expression for each component of the VEV of the energy-momentum tensor is long one. However, we can obtain simpler expressions for $\nu = 1/2$, which corresponds to a conformally coupled massless field. Below we present the expressions for the topological part in the VEV of the energy-momentum tensor for this case.

First we write down the diagonal components in the 4-dimensional AdS spacetime (no summation over μ),

$$\langle T_\mu^\mu \rangle_c = \frac{(a\rho)^{-4}}{96\pi^2} \sum_{k=1}^{q-1} \frac{f_{4,\mu}(\rho s_k, s_k)}{s_k^4 (1+\rho^2 s_k^2)^3} \ , \tag{68}$$

with

$$\begin{aligned} f_{4,0}(u,v) &= -u^6 + \left(9 - 8v^2\right) u^4 + \left(3u^2+1\right)\left(3-2v^2\right) \ , \\ f_{4,1}(u,v) &= -u^6 + \left(9 - 4v^2\right) u^4 + \left(3u^2+1\right)\left(3-2v^2\right) \ , \\ f_{4,2}(u,v) &= -u^6 - \left(27 - 20v^2\right) u^4 - 3\left(3u^2+1\right)\left(3-2v^2\right) \ , \\ f_{4,3}(u,v) &= 3u^6 + \left(9 - 8v^2\right) u^4 + \left(3u^2+1\right)\left(3-2v^2\right) \ , \end{aligned} \tag{69}$$

being $\rho = r/z$. As to the off-diagonal component, we have

$$\langle T_3^1 \rangle_c = -\frac{\rho}{24\pi^2 a^4} \sum_{k=1}^{q-1} \frac{s_k^2}{\left(1+\rho^2 s_k^2\right)^3} \ . \tag{70}$$

For the 6-dimensional case, the VEV of the energy-momentum tensor reads (no summation over μ),

$$\langle T_\mu^\mu \rangle_c = \frac{(a\rho)^{-6}}{640\pi^3} \sum_{k=1}^{q-1} \frac{f_{6,\mu}(\rho s_k, s_k)}{s_k^6 (1+\rho^2 s_k^2)^4} \ , \tag{71}$$

with

$$\begin{aligned} f_{6,l}(u,v) &= -u^8 + 2\left(10 - 9v^2\right) u^6 + (6u^4 + 4u^2 + 1)\left(5 - 4v^2\right) \ , \\ f_{6,1}(u,v) &= -u^8 + 4\left(5 - 3v^2\right) u^6 + (6u^4 + 4u^2 + 1)\left(5 - 4v^2\right) \ , \\ f_{6,2}(u,v) &= -u^8 - 4\left(25 - 21v^2\right) u^6 - 5(6u^4 + 4u^2 + 1)\left(5 - 4v^2\right) \ , \\ f_{6,3}(u,v) &= 5u^8 + 2\left(10 - 9v^2\right) u^6 + (6u^4 + 4u^2 + 1)\left(5 - 4v^2\right) \ , \end{aligned} \tag{72}$$

$l = 0, 4, 5$, for diagonal components, and

$$\langle T_3^1 \rangle_c = -\frac{3\rho}{320\pi^3 a^6} \sum_{k=1}^{q-1} \frac{s_k^2}{\left(1+\rho^2 s_k^2\right)^4} \ , \tag{73}$$

for the off-diagonal component. As it has been mentioned before, all but off-diagonal component diverge for $\rho \to 0$.

Figure 2 presents the dependence of the string-induced part in the VEV of the energy density as a function of the proper distance from the string for $D = 4$ minimally (full curves) and conformally (dashed curves) massless scalar fields. The numbers near the curves correspond to the values of the parameter q. The behavior for large values ρ is plotted separately in the inset to show that for both cases of minimally and conformally coupled fields the energy density goes to zero being positive.

In figure 3 we plot the string-induced part in the energy density versus the parameter q for a fixed value of the distance from the string corresponding to $\rho = 1$. The full and dashed curves correspond to minimally and conformally coupled massless fields in 4-dimensional AdS spacetime. For the conformal coupling the energy density is negative, whereas for the minimal coupling it is positive for $q < 5.6$ and is negative for larger values of q.

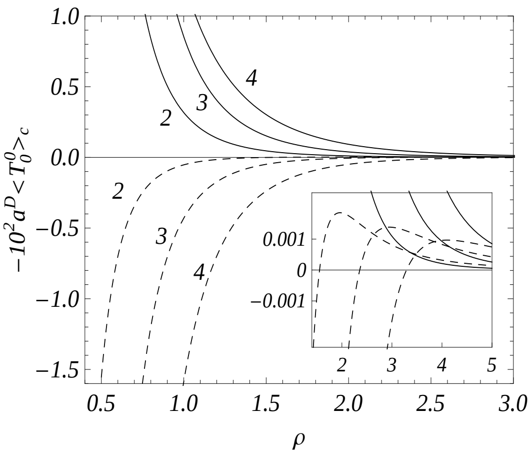

Figure 2: The string-induced part in the VEV of the energy density as a function of the proper distance from the string (measured in units of the AdS curvature radius) for $D = 4$ minimally (full curves) and conformally (dashed curves) massless scalar fields. The numbers near the curves correspond to the values of the parameter q.

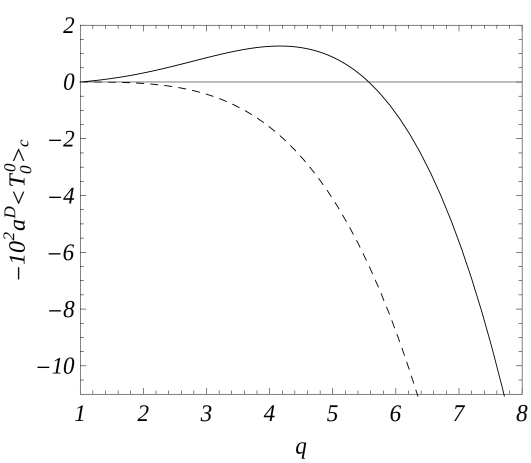

Figure 3: The string-induced part in the VEV of the energy density as a function of the parameter q for fixed proper distance from the string corresponding to $\rho = 1$. The full and dashed curves are for minimally and conformally coupled $D = 4$ massless scalar fields.

5 Conclusion

In this paper we have calculated the VEV of the field squared and the energy-momentum tensor associated with a scalar field in a D-dimensional AdS spacetime in the presence of an idealized cosmic string. In fact, because the calculations of the VEVs in a purely AdS spacetime have been done by several authors, here we were mainly interested in the topological parts in the VEVs induced by the presence of the string. These objective were satisfactorily attained because, for this system, the Wightman function could be expressed as the sum of two terms: the first one due to the AdS background itself in the absence of string, and the second one induced by the presence of the string. This allowed us to write the VEVs as the sum of two contributions following the same structure of the Wightman function. Moreover, because the presence of the string does not modify the curvature of the AdS background, all the divergences presented in the calculations of $\langle\phi^2\rangle$ and $\langle T_\nu^\mu\rangle$, appear only in the contributions due the purely AdS space. So the contributions induced by the string do not require renormalization. All of them are automatically finite for points outside the string. By using (16), we were able to express the Wightman function in a compact form, Eq. (19), allowing us to analyze the VEVs of the field squared and the energy-momentum tensor in a systematic way.

The VEV of the field squared is presented in the decomposed form, Eq. (31), where the part induced by the string is given by the expression (32). For a conformally coupled massless scalar field this expression is conformally related to the corresponding quantity for the geometry of a cosmic string in flat spacetime with an additional flat boundary with Dirichlet boundary condition on it (see (35)). At small proper distances from the string, compared with the AdS curvature radius, the leading term coincides with the corresponding expression for the VEV of the field squared in the geometry of a string in background of flat spacetime, where the distance from the string is replaced by the proper distance ar/z. In this limit the VEV behaves as $(ar/z)^{2-D}$. For a fixed value of the radial coordinate this corresponds to points near the AdS horizon. At large distances from the string, the topological part in the VEV of the field squared decays as $(ar/z)^{1-2\nu-D}$ with the parameter ν defined by (8). In this regime the decay with the distance is power-law for both massless and massive fields. It is worth to note that for a string in background of flat spacetime the decay of the VEV at large distances is power-law for a massless field and exponential for a massive field. We see that the curvature of the background spacetime has an essential influence on the VEVs at distances larger than the curvature radius of the background spacetime. The expression for the VEV of the field squared takes a simpler form, Eq. (42), for integer values of the parameter q related to the angle deficit in the cosmic string geometry. In particular, for a conformally coupled massless field one has the expression (43).

Similar to the case of the field squared, the VEV of the energy-momentum tensor is decomposed as the sum of the pure AdS and string-induced parts. The presence of the string breaks the maximal symmetry of AdS spacetime and the vacuum energy-momentum tensor is non-diagonal. The diagonal components are given by the expressions (52) and the non-zero off-diagonal component is presented by the expression (55). We have explicitly checked that the topological part in the VEV of the energy-momentum tensor obeys the trace relation (57) and the covariant conservation equation. In particular, this part is traceless for a conformally coupled massless scalar field. The trace anomaly in the total energy-momentum tensor is contained in the pure AdS part only. Relatively simple expressions for the VEV of the energy-momentum tensor are obtained near the string and at large distances. Near the string, the topological parts in the diagonal components scale as the inverse D-th power of the proper distance from the string, whereas the off-diagonal component scales as the inverse $(D-1)$-th power of the distance. For a conformally coupled massless scalar field the off-diagonal component vanishes on the string

linearly with the distance. The pure AdS part in the vacuum energy-momentum tensor is a constant everywhere and near the string the total VEV is dominated by the topological part. In this region the back-reaction of the string-induced quantum effects on the bulk geometry is important. At large distances from the string, compared with the AdS curvature radius, the decay of the topological part with the distance is power-law for both massless and massive fields. The leading terms in the corresponding asymptotic expansions are given by expressions (64)-(66). These expressions describe also the behavior of the VEVs for points near the AdS boundary. For a fixed value of the radial coordinate, the string-induced parts vanish on the AdS boundary as $z^{2\nu+D-1}$. The general formulas for the VEV of the energy-momentum tensor are simplified in the special case of integer values for the parameter q. The corresponding expressions are given by Eq. (67). These expressions are further simplified for a conformally coupled massless scalar field. We present explicit formulas for the energy momentum-tensor in spacetime dimensions $D = 4$ and $D = 6$.

In a way similar to that described above, we can evaluate one-loop quantum effects induced by the string in AdS spacetime in the presence of branes. For the corresponding geometry with two branes in background of pure AdS spacetieme (employed in Randall-Sundrum type braneworld models) the VEVs of the field squared and the energy-momentum tensor are investigated in [44, 45] (see also [46, 47] for the models with compact internal spaces). In the presence of the cosmic string, the mode functions in the region between the branes are given by expression (6), with the function $J_\nu(\lambda z)$ replaced by the linear combination of the Bessel and Neumann functions with the same argument. The eigenvalues for λ are determined from the boundary conditions on the branes and they are expressed in terms of the zeros of the cross-product of the Bessel and Neumann functions. The mode-sum for the Wightman function contains summation over these zeros. By applying to this sum the generalized Abel-Plana formula from [48], we can present the VEVs as the sum of boundary-free AdS parts, investigated in the present paper, and the parts induced by the branes. The investigation for the latter will be presented elsewhere. An interesting application of the results presented in this paper would be the investigation of the corresponding effects in the boundary conformal field theory by using the AdS/CFT correspondence.

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